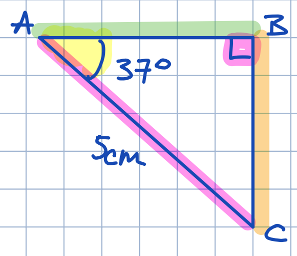
Chapitre 7 : Trigonométrie

$\cos(\text{Angle}) = \dfrac{\text{côté adjacent}}{\text{Hypoténuse}}$

I) Rappel :



Calculer AB :

Dans le triangle ABC rectangle en B

H

$\cos(\widehat{BAC}) = \dfrac{\text{coté adjacent à } \widehat{BAC}}{\text{Hypoténuse}}$ T

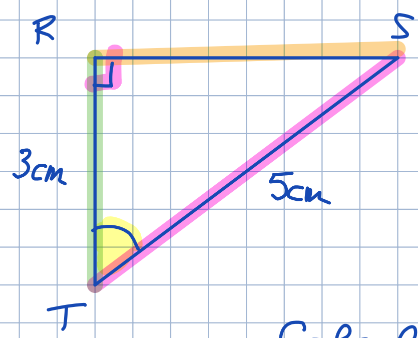
C

$\cos(\widehat{BAC}) = \dfrac{AB}{AC}$

$\dfrac{\cos(37°)}{1} = \dfrac{AB}{5}$

$AB = \dfrac{5 \times \cos(37°)}{1}$

$AB \simeq 4 \text{ cm}$ à 0,1 près



Calculer $\widehat{RTS}$:

H

Dans le triangle RST rectangle en R

T

$\cos(\widehat{RTS}) = \dfrac{\text{côté adjacent à } \widehat{RTS}}{\text{Hypoténuse}}$

C

$\cos(\widehat{RTS}) = \dfrac{RT}{TS}$

$\cos(\widehat{RTS}) = \dfrac{3}{5}$

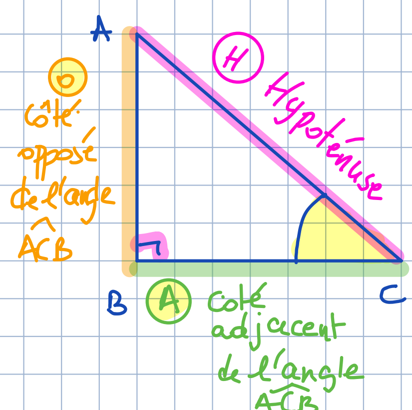
Calculatrice :

$$\boxed{\widehat{RTS} \underset{\cos^{-1}\ (\text{arccos})}{\overset{\cos}{\rightleftarrows}} \cos(\widehat{RTS})}$$

$\widehat{RTS} = \arccos\left(\dfrac{3}{5}\right)$

$\widehat{RTS} \simeq 53°$ à 1 degré près

Autres formules :



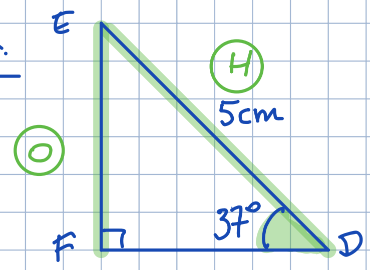
Dans le triangle rectangle ABC

* $\cos(\widehat{ACB}) = \dfrac{\text{côté adjacent de } \widehat{ACB}}{\text{Hypoténuse}}$ (cosinus) CAH

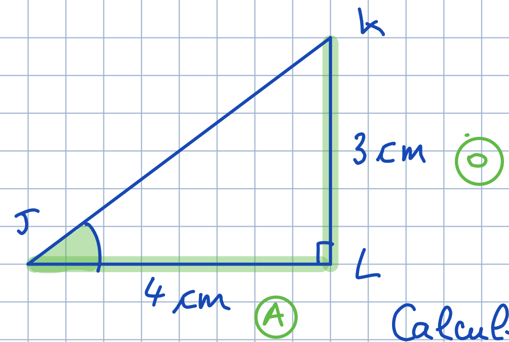
* $\sin(\widehat{ACB}) = \dfrac{\text{côté opposé de } \widehat{ACB}}{\text{Hypoténuse}}$ (sinus) SOH

* $\tan(\widehat{ACB}) = \dfrac{\text{côté opposé de } \widehat{ACB}}{\text{côté adjacent de } \widehat{ACB}}$ (tangente) TOA

Exemple:



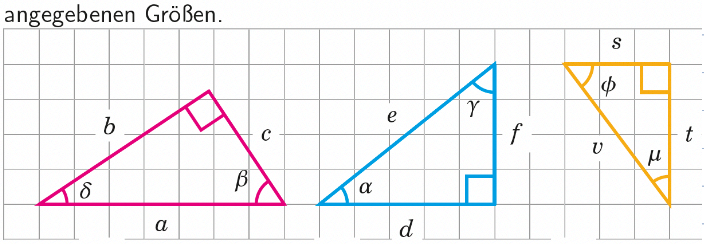
Calculer FE



Calculer $\widehat{KJL}$

■ **Übung 3 : Lücken**

Vervollständige die Gleichungen für das jeweilige Dreieck mit den angegebenen Größen.



$\sin\delta = \dfrac{c}{a}$ $\qquad\cos\alpha = \dfrac{d}{e}$ $\qquad\tan\phi = \dfrac{t}{s}$

$\sin\beta = \dfrac{b}{a}$ $\qquad\cos\gamma = \dfrac{f}{e}$ $\qquad\tan\mu = \dfrac{s}{t}$

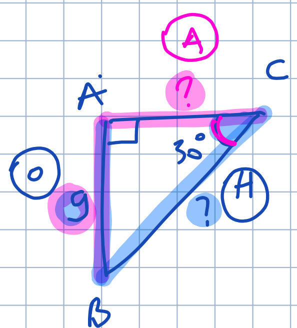
$\tan\beta = \dfrac{b}{c}$ $\qquad\tan\alpha = \dfrac{f}{d}$ $\qquad\sin\varphi = \dfrac{t}{v}$

α alpha
β beta
γ gamma
φ phi
μ mu
δ delta.

■ **Übung 4 : Seitenlänge**

Für jedes Dreieck bestimme die fehlende Seitenlänge.

- ▼ ABC ist rechtwinklig in A, sodass AB = 9 cm und $\widehat{ACB} = 30°$.
- ▲ DEF ist rechtwinklig in E, sodass EF = 7 cm und $\widehat{FDE} = 65°$.
- ▼ IJK ist rechtwinklig in I, sodass JI = 10 cm und $\widehat{IKJ} = 55°$.
- ▲ LMN ist rechtwinklig in N, sodass LN = 11 cm und $\widehat{LMN} = 33°$.
- ▼ RST ist rechtwinklig in T, sodass RT = 13 cm und $\widehat{RST} = 70°$.



CAH
SOH
TOA

$$\frac{\sin(30°)}{1} = \frac{9}{BC}$$

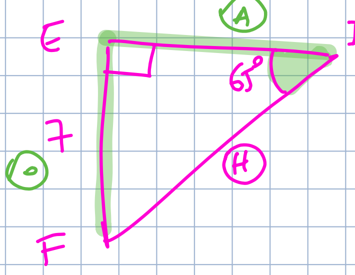
$$BC = \frac{9 \times 1}{\sin(30°)}$$

$$BC = 18 \text{ cm}$$

$$\tan(30°) = \frac{9}{AC}$$

$$AC = \frac{9}{\tan(30°)}$$

$AC \approx 15{,}6$ cm auf 0,1 gerundet.



$\tan(65°) = \frac{7}{ED}$

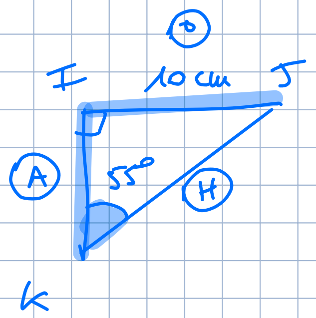
$ED = \frac{7}{\tan(65°)}$

$ED \approx 3{,}3$ cm auf 0,1 gerundet

$\sin(65°) = \frac{7}{FD}$

$FD = \frac{7}{\sin(65°)}$

$FD \approx 7{,}7$ cm auf 0,1 gerundet



$\tan(55°) = \frac{10}{IK}$

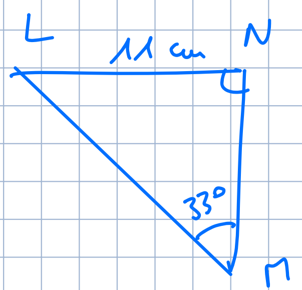
$IK = \frac{10}{\tan(55°)}$

$IK \approx$

$\sin(55°) = \frac{10}{KJ}$

$KJ = \frac{10}{\sin(55°)}$

$KJ \approx$



$\tan(33°) = \frac{11}{MN}$

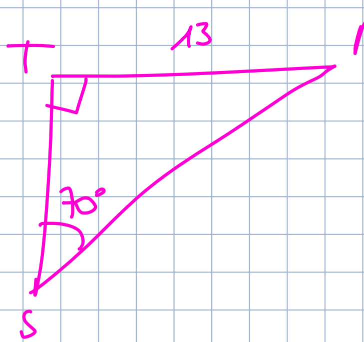
$MN = \frac{11}{\tan(33°)}$

$MN \approx$

$\sin(33°) = \frac{11}{LM}$

$LM = \frac{11}{\sin(33°)}$

$LM \approx$



$\tan(70°) = \frac{13}{TS}$

$TS = \frac{13}{\tan(70°)}$

$TS \approx$

$\sin(70°) = \frac{13}{RS}$

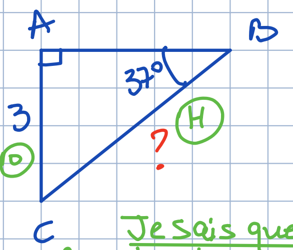
$RS = \frac{13}{\sin(70°)}$

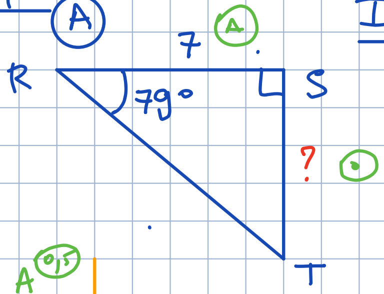
$RS \approx$

arrondir à 0,1 près

Interrogation écrite :

A





Je sais que :
ABC est rectangle en A 0,5

J'utilise : $\sin(\widehat{ABC}) = \dfrac{AC}{BC}$ 1

$\sin(37°) = \dfrac{3}{BC}$

Donc : $BC = \dfrac{3}{\sin(37°)}$ 0,5

≈ 5 0,5

à 0,1 près

Je sais que :
RST est rectangle en S

J'utilise : $\tan(\widehat{SRT}) = \dfrac{ST}{RS}$

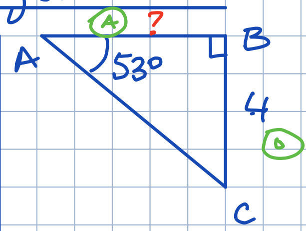
$\tan(79°) = \dfrac{ST}{7}$

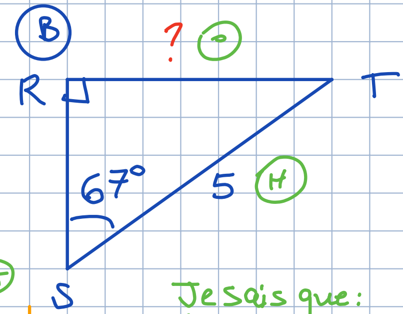
Donc : $ST = 7 \times \tan(79°)$

≈ 36

à 0,1 près

B





Je sais que :
ABC est rectangle en B 0,5

J'utilise : $\tan(\widehat{BAC}) = \dfrac{BC}{AB}$ 1

$\tan(53°) = \dfrac{4}{AB}$

Donc : $AB = \dfrac{4}{\tan(53°)}$ 0,5

≈ 3 0,5

à 0,1 près

Je sais que :
RST est rectangle en R

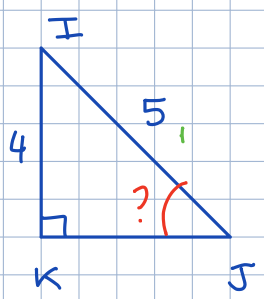
J'utilise : $\sin(\widehat{RST}) = \dfrac{RT}{ST}$

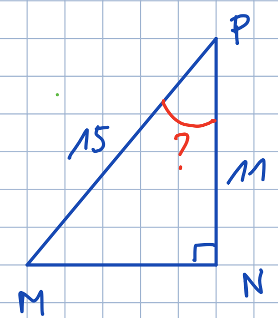
$\sin(67°) = \dfrac{RT}{5}$

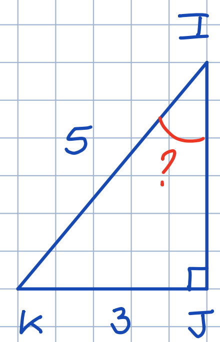
Donc : $RT = 5 \times \sin(67°)$

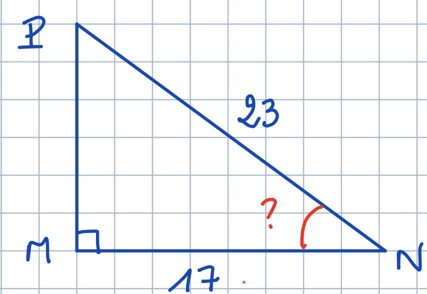
$\approx 4,6$

à 0,1 près



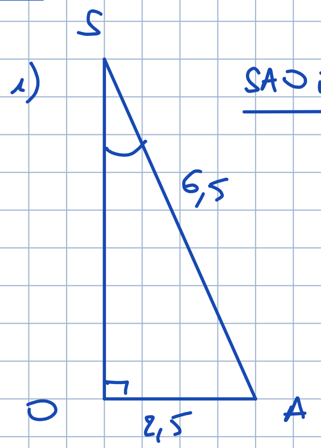






Üb 8 S 40 Üb 7-8 S 39

1)



SAO ist rechtwinklig in O

2) Im Dreieck SAO rechtwinklig in O

Ich benutze den Satz von Pythagoras.

$SA^2 = SO^2 + OA^2$

daraus folgt: $SO^2 = SA^2 - OA^2$

$SO^2 = 6{,}5^2 - 2{,}5^2$

$SO^2 = 36$

$SO = 6\,cm$

3) $\text{Volumen} = \frac{\pi \times R^2 \times H}{3} = \frac{\pi \times 2{,}5^2 \times 6}{3} = 12{,}5\pi \approx 39{,}270\ cm^3$ auf mm^3 gerundet.

4) $0{,}955\,g/cm^3 =$

$39{,}270 \times 0{,}955 \approx 37{,}5\,g$.

5) Im Dreieck ASO rechtwinklig in O

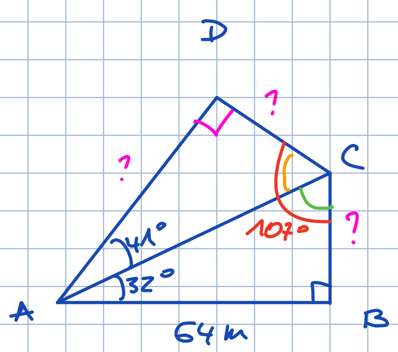
Ich benutze: $\cos(\widehat{ASO}) = \frac{SO}{SA}$

$\cos(\widehat{ASO}) = \frac{6}{6{,}5}$

$\widehat{ASO} = \arccos\left(\frac{6}{6{,}5}\right)$

$\widehat{ASO} \approx 23°$ auf Einer gerundet.

Üb 7:



Ich rechne $\widehat{ACB}$: In einem Dreieck die Winkelsumme beträgt 180°

$\widehat{ACB} = 180° - \widehat{CAB} - \widehat{CBA} = 180° - 32 - 90 = 58°$

Ich rechne $\widehat{DCA}$: $\widehat{DCA} = \widehat{DCB} - \widehat{ACB} = 107° - 58° = 49°$

Ich rechne $\widehat{CDA}$ Im Dreieck ADC hat man -

$\widehat{ACD} + \widehat{CDA} + \widehat{DAC} = 180°$

$\widehat{CDA} = 180 - \widehat{DAC} - \widehat{ACD} = 180 - 41° - 49°$

$\boxed{\widehat{CDA} = 90°}$

Daraus folgt: ADC ist rechtwinklig in D.

Ich rechne BC: Im Dreieck ABC rechtwinklig in B

$\tan(\widehat{CAB}) = \frac{CB}{AB}$ Also $\tan(32°) = \frac{CB}{64}$

$CB = 64 \times \tan(32°) \simeq 40$ m.

Ich rechne AC:

$\cos(\widehat{CAB}) = \frac{AB}{AC}$ Also $\cos(32°) = \frac{64}{AC}$

$AC = \frac{64}{\cos(32°)} \simeq 75{,}5$ m

Ich rechne DC Im Dreieck ACD rechtwinklig in D

$\sin(\widehat{DAC}) = \frac{DC}{AC}$ Also $\sin(41°) \simeq \frac{DC}{75{,}5}$

$DC \simeq 75{,}5 \times \sin(41°)$

$DC \simeq 49{,}5$ m.

$\cos(\widehat{DAC}) = \frac{AD}{AC}$ Also $\cos(41°) = \frac{AD}{75{,}5}$

$AD \simeq 75{,}5 \times \cos(41°)$

$AD \simeq 57$ m.

Die Umfang beträgt: $AB + BC + CD + DA$

$\simeq 64 + 40 + 49{,}5 + 57$

$\simeq 210{,}5$ m.