Chapitre 5 : Intégration (suite)

Exercices :

Ex1 : 2) $f(x) = \sqrt{e^x}$ $\qquad F(x) = 2\sqrt{e^x}$ $\qquad (\sqrt{u})' = \dfrac{u'}{2\sqrt{u}}$

$$F'(x) = \cancel{2} \times \frac{e^x}{\cancel{2}\sqrt{e^x}} = \frac{e^x}{\sqrt{e^x}} = \frac{\sqrt{e^x} \times \cancel{\sqrt{e^x}}}{\cancel{\sqrt{e^x}}} = \sqrt{e^x} = f(x)$$

$u = e^x \quad u' = e^x$

F est bien une primitive de f.

Ex2 : 1) $f_1(x) = 4x^3 + 3x^2 + 2x + 1$ $\qquad f(x) = x^n$

$F_1(x) = \cancel{4}\dfrac{x^4}{\cancel{4}} + \cancel{3}\dfrac{x^3}{\cancel{3}} + \cancel{2}\dfrac{x^2}{\cancel{2}} + x$ $\qquad F(x) = \dfrac{x^{n+1}}{n+1}$

$F_1(x) = x^4 + x^3 + x^2 + x + k,\ k \in \mathbb{R}$.

2) $f_2(x) = 6x^5 + 4x^3 - 1$

$F_2(x) = \cancel{6}\dfrac{x^6}{\cancel{6}} + \cancel{4}\dfrac{x^4}{\cancel{4}} - x + k,\ k \in \mathbb{R}$

$F_2(x) = x^6 + x^4 - x + k,\ k \in \mathbb{R}$.

3) $f_3(x) = x^3 + x^2 + x + 1$

$F_3(x) = \dfrac{x^4}{4} + \dfrac{x^3}{3} + \dfrac{x^2}{2} + x \quad + k \in \mathbb{R}$

4) $f_4(x) = 4x^7 - x^6 - \dfrac{2}{3}x - 5 \times 1$

$F_4(x) = 4\dfrac{x^8}{8} - \dfrac{x^7}{7} - \dfrac{2}{3}\dfrac{x^2}{2} - 5x \quad + k \in \mathbb{R}$

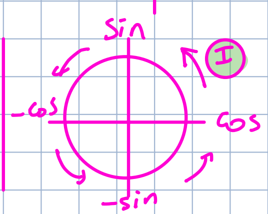
$F_4(x) = \dfrac{x^8}{2} - \dfrac{x^7}{7} - \dfrac{x^2}{3} - 5x \quad + k \in \mathbb{R}$.

5) $f_5(x) = \dfrac{1}{2\sqrt{x}} + 9$ $\qquad f(x) = \dfrac{1}{2\sqrt{x}}$

$F_5(x) = \sqrt{x} + 9x \quad + k \in \mathbb{R}$ $\qquad F(x) = \sqrt{x}$

6) $f_6(x) = \sin(x) - 3\cos(x)$

$F_6(x) = -\cos(x) - 3\sin(x)$



7) $f_7(x) = \frac{2}{x} = 2 \times \frac{1}{x}$ $x \in \mathbb{R}_+^* \iff x > 0$

$F_7(x) = 2 \times \ln(x) = 2\ln(x)$

$f(x) = \frac{1}{x}$

$F(x) = \ln(x)$

$x > 0$

8) $f_8(x) = -\frac{1}{x^2} + \frac{1}{x} - e^x$, $x \in \mathbb{R}_+^*$

$F_8(x) = \frac{1}{x} + \ln(x) - e^x + k$, $x \in \mathbb{R}_+^*$
$k \in \mathbb{R}$

$f(x) = e^x$
$F(x) = e^x$

$f(x) = -\frac{1}{x^2}$

$F(x) = \frac{1}{x}$

Ex 3 : ① $\frac{u'}{u} \to \ln(u)$ si $u > 0$

② $\frac{u'}{u^2} \to \frac{-1}{u}$ si $u \neq 0$

③ $u'e^u \to e^u$

Exemple : $f(x) = 2x\,e^{x^2}$

③ $\to u(x) = x^2$

$u'(x) = 2x$

$\boxed{F(x) = e^{x^2}}$

1) $f_1(x) = \frac{2x}{x^2+1}$

$F_1(x) = \ln(x^2+1)$, $x \in \mathbb{R}$.

or $\underbrace{x^2}_{\geq 0} + 1 > 0$

① $u(x) = x^2 + 1$
$u'(x) = 2x$

$\frac{u'}{u} = \frac{2x}{x^2+1}$

2) $f_2(x) = \frac{4x^3}{(x^4+1)^2}$

$F_2(x) = \frac{-1}{x^4+1}$, $x \in \mathbb{R}$

$x^4 + 1 = 0$
$\left.\begin{array}{l} x^4 = -1 \\ x^4 > 0 \end{array}\right)$ impossible

② $u(x) = x^4 + 1$

$u'(x) = 4x^3$

$\frac{u'}{u^2} = \frac{4x^3}{(x^4+1)^2}$

3) $f_3(x) = 3x\,e^{3x^2+1}$

$f_3(x) = \frac{1}{2} \times \underbrace{2 \times 3x}_{6x} e^{3x^2+1}$

$= \frac{1}{2} \times \underbrace{6x\,e^{3x^2+1}}$

$F_3(x) = \frac{1}{2} \times e^{3x^2+1} = \frac{1}{2}\,e^{3x^2+1}$

③ $u'e^u \to$ $u(x) = 3x^2 + 1$

$u'(x) = 3 \times 2x = 6x$

$6x\,e^{3x^2+1}$

3bis) $f_{3B}(x) = 2x\, e^{2x^2+1}$ ③ $u'e^u \rightarrow$ $u(x) = 2x^2+1$

$u'(x) = 2 \times 2x = 4x$

$= \frac{1}{2} \times \underbrace{2 \times 2x\, e^{2x^2+1}}$

$= \frac{1}{2} \times \underbrace{4x\, e^{2x^2+1}}$

$F_{3B}(x) = \frac{1}{2}\, e^{2x^2+1}$

4) $f_4(x) = \frac{e^x}{(e^x+1)^2}$ ② $\frac{u'}{u^2} \rightarrow$ $u(x) = e^x+1$

$u'(x) = e^x$

$F_4(x) = \frac{-1}{e^x+1}$, $x \in \mathbb{R}$

$e^x + 1 = 0$
$e^x = -1$ impossible car $e^x > 0$

5) $f_5(x) = \frac{e^x}{e^x+3}$ ① $\frac{u'}{u} \rightarrow$ $u(x) = e^x+3$

$u'(x) = e^x$

$F_5(x) = \ln(e^x+3)$, $x \in \mathbb{R}$

$e^x + 3 > 0$
$e^x > -3$ toujours vrai

6) $f_6(x) = \cos(x)\, e^{\sin(x)}$ ③ $u'e^u \rightarrow$ $u(x) = \sin(x)$

$u'(x) = \cos(x)$

$F_6(x) = e^{\sin(x)}$, $x \in \mathbb{R}$

Ex4: $f(x) = \frac{x}{x^2+4}$ ① $\frac{u'}{u} \rightarrow$ $u(x) = x^2+4$

$u'(x) = 2x$

$= \frac{1}{2} \times 2 \times \frac{x}{x^2+4}$

$= \frac{1}{2} \times \boxed{\frac{2x}{x^2+4}}$

$\boxed{F(x) = \frac{1}{2} \ln(x^2+4) + k}$, avec $k \in \mathbb{R}$

$x^2+4 > 0$
$x^2 > -4$ toujours vrai

Remarque: toutes les primitives de $f(x) = \frac{x}{x^2+4}$ sont de la forme $F_k(x) = \frac{1}{2}\ln(x^2+4) + k$
$k \in \mathbb{R}$

Par contre il n'y en a qu'une qui vaut y_0 pour $x = x_0$

__Exemple :__ Si on veut que $F_k(0) = 1$

il faut que $\left.\begin{array}{l} F_k(0) = \frac{1}{2}\ln(0^2+4) + k = \frac{1}{2}\ln(4) + k \\ F_k(0) = 1 \end{array}\right\}$

$$\frac{1}{2}\ln(4) + k = 1$$

$$k = 1 - \frac{1}{2}\ln(4)$$

donc l'unique primitive de f qui vaut 1 pour $x=0$

est $F(x) = \frac{1}{2}\ln(x^2+4) + 1 - \frac{1}{2}\ln(4)$

__Résumé__ Ⓐ

$F(x) \longrightarrow f(x) \longrightarrow f'(x)$

$\frac{x^3}{3} \longrightarrow x^2 \longrightarrow 2x$

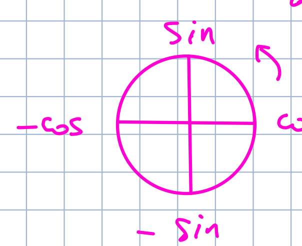
Primitive $\longrightarrow$ Fonction $\longrightarrow$ Dérivée

__Formules essentielles :__

$x^n \longrightarrow \frac{x^{n+1}}{n+1}$

$e^x \longrightarrow e^x$

$e^{ax} \longrightarrow \frac{e^{ax}}{a}$



Ⓑ __Formules composées :__

$u' \times u^n \longrightarrow \frac{u^{n+1}}{n+1}$

__Ex :__ ① $f(x) = 2x\,(x^2+1)^4 = u' \times u^4 \longrightarrow \frac{u^5}{5}$

$u = x^2+1$

$u' = 2x$

$F(x) = \frac{(x^2+1)^5}{5}$

② $f(x) = \frac{3}{3} \times x^2 e^{x^3}$

$u'e^u \longrightarrow e^u$

$3x^2 e^{x^3} \longrightarrow e^{x^3}$

$u = x^3$

$u' = 3x^2$

$f(x) = \frac{1}{3} \times \underbrace{3x^2 e^{x^3}} \longrightarrow F(x) = \frac{1}{3} \times e^{x^3} = \frac{1}{3} e^{x^3}$

③ $f(x) = \frac{x}{x^2+4} = \frac{1}{2} \boxed{\frac{2x}{x^2+4}}$

$\frac{u'}{u} \longrightarrow \ln(u) \quad u > 0$

$\frac{u'}{u} = \frac{2x}{x^2+4} \longrightarrow \ln(x^2+4)$

$u = x^2 + 4 > 0$

$u' = 2x$

$F(x) = \frac{1}{2}\ln(x^2+4)$

$F'(x) = f(x)$

Ⓒ $f(x) = \frac{x}{x^2+4}$ $\quad F(x) = \frac{1}{2}\ln(x^2+4) + k \quad , k \in \mathbb{R}$

Trouver F la primitive de f telle que $\underline{F(0) = 0}$

$$F(x) = \frac{1}{2} \ln(x^2+4) + k$$

$$F(0) = \frac{1}{2} \ln(0^2+4) + k$$
$$= \frac{1}{2} \ln(4) + k$$

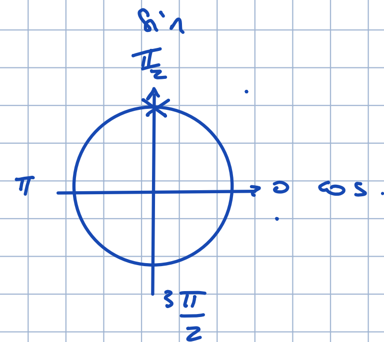
$$\frac{1}{2} \ln(4) + k = 0$$
$$k = -\frac{1}{2} \ln(4)$$

Donc $\underline{\underline{la}}$ Primitive F telle que $F(0)=0$ est :

$$F(x) = \frac{1}{2} \ln(x^2+4) - \frac{1}{2} \ln(4)$$

__Ex5:__ 1) $f(x) = \cos(x)$

$$\begin{cases} F(x) = \sin(x) + k, \; k \in \mathbb{R} \\ F(\frac{\pi}{2}) = 0 \end{cases}$$



$$F\left(\frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) + k$$
$$F\left(\frac{\pi}{2}\right) = 1 + k$$

$$1 + k = 0$$
$$k = -1$$

donc $F(x) = \sin(x) - 1$

2) $f(x) = e^{3x+1}$

$u = 3x+1$

$u' = 3$

$u' e^u \rightarrow e^u$

$3 e^{3x+1} \rightarrow e^{3x+1}$

$f(x) = \frac{1}{3} \times 3 e^{3x+1}$

$F(x) = \frac{1}{3} \times e^{3x+1} + k, \; k \in \mathbb{R}$

$$\begin{cases} F(x) = \frac{1}{3} e^{3x+1} + k \\ F(-1) = 0 \end{cases}$$

$$F(-1) = \frac{1}{3} e^{3\times(-1)+1} + k = \frac{1}{3} e^{-2} + k$$

$$\frac{1}{3} e^{-2} + k = 0$$
$$k = -\frac{1}{3} e^{-2}$$

$$F(x) = \frac{1}{3} e^{3x+1} - \frac{1}{3} e^{-2}$$

3) $f(x) = x e^{-x^2}$

$u'e^{u} \longrightarrow e^{u}$

$-2x e^{-x^2} \longrightarrow e^{-x^2}$

$u = -x^2$

$u' = -2x$

$$f(x) = -\frac{1}{2} \times \left(-2x e^{-x^2}\right)$$

$$\begin{cases} F(x) = -\frac{1}{2} \times e^{-x^2} + k, & k \in \mathbb{R} \\ F\left(\sqrt{\ln 2}\right) = 1 \end{cases}$$

$$F\left(\sqrt{\ln 2}\right) = -\frac{1}{2} e^{-\left(\sqrt{\ln 2}\right)^2} + k$$

$$= -\frac{1}{2} e^{-\ln(2)} + k$$

$$= -\frac{1}{2} \frac{1}{e^{\ln(2)}} + k$$

$$= -\frac{1}{2} \times \frac{1}{2} + k$$

$10^{-2} = \frac{1}{10^2}$

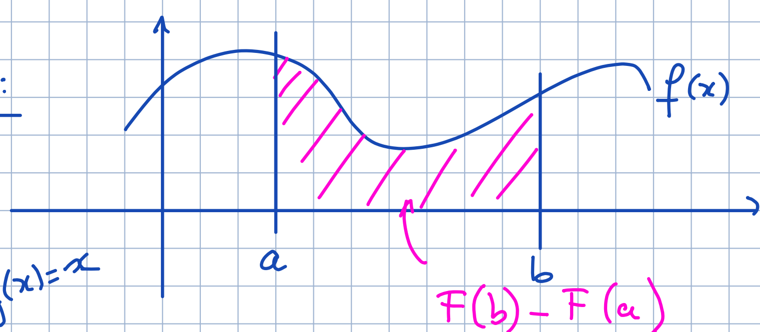
$e^{-\ln(2)} = \frac{1}{e^{\ln(2)}}$

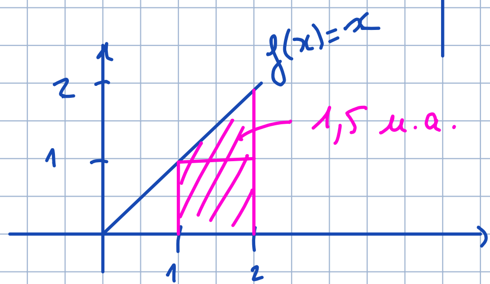
$$\left.\begin{aligned} F\left(\sqrt{\ln 2}\right) &= -\frac{1}{4} + k \\ F\left(\sqrt{\ln 2}\right) &= 1 \end{aligned}\right\} \quad -\frac{1}{4} + k = 1$$

$$k = 1 + \frac{1}{4} = \frac{5}{4}$$

donc $F(x) = -\frac{1}{2} e^{-x^2} + \frac{5}{4}$

Ex7 Intégrales :





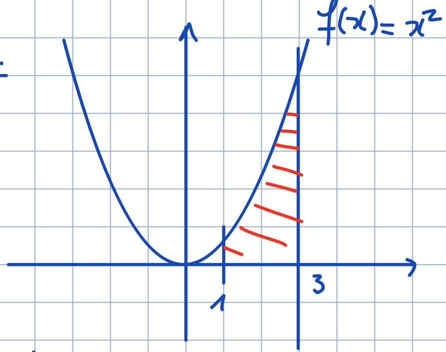
$$\int_1^2 f(x)\,dx = F(2) - F(1)$$

$$\int_1^2 x\,dx = F(2) - F(1) = \frac{2^2}{2} - \frac{1^2}{2}$$

$f(x) = x \quad F(x) = \frac{x^2}{2}$

$$= \frac{4}{2} - \frac{1}{2} = \frac{3}{2} = 1{,}5$$

Ex:



$$\int_1^3 x^2 \, dx = F(3) - F(1) = \frac{3^3}{3} - \frac{1^3}{3} = \frac{27}{3} - \frac{1}{3}$$

$$= \frac{26}{3} \text{ u.a.}$$

$$\simeq 8{,}67 \text{ u.a.}$$

$f(x) = x^2 \qquad F(x) = \frac{x^3}{3}$

Ex7 : $\int_{-1}^{1} (t^2 + 4t + 3) \, dt = F(1) - F(-1)$

$f(t) = t^2 + 4t + 3 \qquad F(t) = \frac{t^3}{3} + 4\frac{t^2}{2} + 3t = \frac{t^3}{3} + 2t^2 + 3t$

$t^2 \longrightarrow \frac{t^3}{3} \qquad x^n \longrightarrow \frac{x^{n+1}}{n+1}$

$t^1 \longrightarrow \frac{t^2}{2}$

$3 \longrightarrow 3t$

$$\int_{-1}^{1} (t^2+4t+3) \, dt = \frac{1^3}{3} + 2 \times 1^2 + 3 \times 1 - \left(\frac{(-1)^3}{3} + 2 \times (-1)^2 + 3 \times (-1) \right)$$

$$= \frac{1}{3} + 2 + 3 - \left(\frac{-1}{3} + 2 - 3 \right)$$

$$= \frac{1}{3} + \cancel{2} + 3 + \frac{1}{3} - \cancel{2} + 3$$

$$= \frac{2}{3} + 6$$

$$= \frac{20}{3}$$

$\simeq 6{,}67$ à 10^{-2} près.
u.a.

b) $\int_1^2 \left(\frac{1}{2t} - \frac{3}{t^2} \right) dt$

$f(t) = \frac{1}{2t} - \frac{3}{t^2} = \frac{1}{2} \times \frac{1}{t} - 3 \times \frac{1}{t^2}$

$F(t) = \frac{1}{2} \times \ln(t) - 3 \times \left(\frac{-1}{t} \right) = \frac{1}{2} \ln(t) + \frac{3}{t}$, $t > 0$

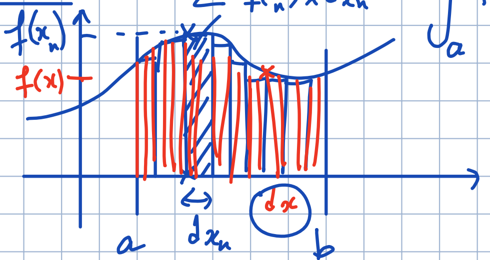
$$\int_1^2 \left(\frac{1}{2t} - \frac{3}{t^2} \right) dt = F(2) - F(1) = \frac{1}{2} \ln(2) + \frac{3}{2} - \left(\frac{1}{2} \ln(1) + \frac{3}{1} \right)$$

$$= \frac{1}{2} \ln(2) + \frac{3}{2} - 3 = \frac{1}{2} \ln(2) - \frac{3}{2}$$

<u>Remarque:</u> $\int_a^b f(x)\,dx = F(b) - F(a)$

$f(x) \xrightarrow{\text{Primitive}} F(x)$

<u>Notation:</u> $\sum f(x_n) \times dx_n = \int_a^b f(x)\,dx$



c) $\int_{\ln(2)}^{\ln(3)} e^x\,dx = e^{\ln(3)} - e^{\ln(2)} = 3 - 2 = 1$ u.a.

$f(x) = e^x$

<u>Primitive:</u> $F(x) = e^x$

d) $\int_1^2 \sqrt{t}\,dt = \frac{2}{3} \times 2\sqrt{2} - \frac{2}{3} \times 1\sqrt{1} = \frac{4\sqrt{2}}{3} - \frac{2}{3}$

$(\sqrt{t})^2 = t$

$(t^{\frac{1}{2}})^2 = t^{\frac{1}{2}\times 2} = t^1 = t$

$f(t) = \sqrt{t} = t^{\frac{1}{2}}$ $\qquad x^n \longrightarrow \frac{x^{n+1}}{n+1}$

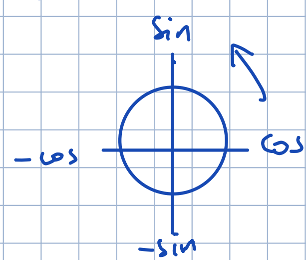
$F(t) = \frac{2}{3} t\sqrt{t}$ $\qquad t^{\frac{1}{2}} \rightarrow \frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} = \frac{t^{\frac{1}{2}} \times t^1}{3/2} = \frac{t\sqrt{t}}{\frac{3}{2}} = \frac{2}{3} t\sqrt{t}$

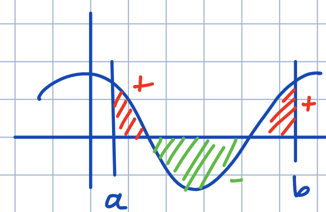
e) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\cos(x) - \sin(x))\,dx = \sin\left(\frac{\pi}{2}\right) + \cos\left(\frac{\pi}{2}\right) - \left[\sin\left(\frac{\pi}{6}\right) + \cos\left(\frac{\pi}{6}\right)\right] = 1 + 0 - \left[\frac{1}{2} + \frac{\sqrt{3}}{2}\right]$

$= 1 - \frac{1}{2} - \frac{\sqrt{3}}{2} = \frac{1-\sqrt{3}}{2}$

$f(x) = \cos(x) - \sin(x)$

$F(x) = \sin(x) + \cos(x)$





<u>Ex8:</u> (cf Ex3)

a) $\int_0^{\ln(3)} e^{-t}\,dt = -e^{-\ln(3)} - (-e^{-0}) = -e^{-\ln(3)} + 1 = -e^{\ln(\frac{1}{3})} + 1 = -\frac{1}{3} + 1 = \frac{2}{3}$

$f(t) = e^{-t} = e^{-1t}$

$F(t) = \frac{e^{-1t}}{-1} = -e^{-t}$

$e^{ax} \longrightarrow \frac{e^{ax}}{a}$

$-\ln(3) = \ln(3^{-1}) = \ln\left(\frac{1}{3}\right)$

$\ln\left(\frac{1}{a}\right) = -\ln(a)$, $a > 0$

b) $\int_0^2 \frac{x}{(x^2+1)^2}\,dx = -\frac{1}{2}\times\frac{1}{2^2+1} - \left(-\frac{1}{2}\times\frac{1}{0^2+1}\right) = -\frac{1}{10}+\frac{1}{2} = 0{,}4$

$\frac{u'}{u^2} \longrightarrow -\frac{1}{u}$

$f(x) = \frac{1}{2}\times\frac{2x}{(x^2+1)^2}$

$u = x^2+1$

$u' = 2x$

$F(x) = \frac{1}{2}\times\frac{-1}{x^2+1}$

$= -\frac{1}{2}\times\frac{1}{x^2+1}$

c) $\int_0^{-1} \frac{dt}{\sqrt{1-2t}} = \int_0^{-1} \frac{1}{\sqrt{1-2t}}\times dt = -\sqrt{1-2\times(-1)} - \left(-\sqrt{1-2\times 0}\right) = -\sqrt{3}-1$

$f(t) = \frac{1}{\sqrt{1-2t}} = \frac{1}{-2}\times\frac{-2}{\sqrt{1-2t}}$

$\frac{u'}{\sqrt{u}} \longrightarrow 2\sqrt{u}$

$u = 1-2t$

$u' = -2$

$F(t) = \frac{1}{-2}\times 2\sqrt{1-2t}$

$F(t) = -\sqrt{1-2t}$

d) $\int_1^2 \frac{dx}{3x+2} = \frac{1}{3}\ln(3\times 2+2) - \frac{1}{3}\ln(3\times 1+2) = \frac{1}{3}\ln(8) - \frac{1}{3}\ln(5) = \frac{1}{3}\ln\left(\frac{8}{5}\right)$

$f(x) = \frac{1}{3x+2} = \frac{1}{3}\,\frac{3}{3x+2}$

$\frac{u'}{u} \rightarrow \ln(u)$

$u = 3x+2$

$u' = 3$

$F(x) = \frac{1}{3}\ln(3x+2)$

$u > 0$

$3x+2 > 0$

$3x > -2$

$x > -\frac{2}{3}$ ok car $x \in [1;2]$.

(g) (h) (i)

Ex :

① $\displaystyle\int_1^2 x^3\,dx$

① $x^3 \to \dfrac{x^{3+1}}{3+1} = \dfrac{x^4}{4}$

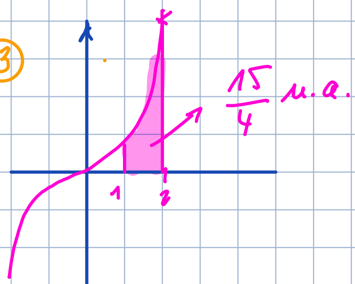
$x^n \to \dfrac{x^{n+1}}{n+1}$

② $\displaystyle\int_1^2 x^3\,dx = \frac{2^4}{4} - \frac{1^4}{4}$

$= \dfrac{16}{4} - \dfrac{1}{4}$

$= \dfrac{15}{4}$

③



② $\displaystyle\int_{e^2}^{e^4} \frac{1}{t}\,dt$

① $\dfrac{1}{t} \quad \ln(t)$

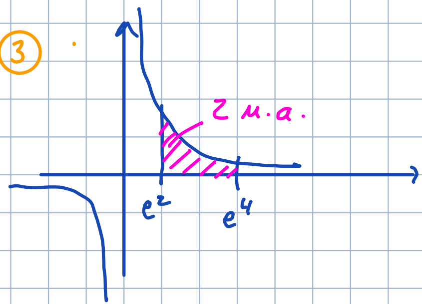
$\dfrac{1}{t} \to \ln(t)$

② $\displaystyle\int_{e^2}^{e^4} \frac{1}{t}\,dt = \ln(e^4) - \ln(e^2)$

$= 4 - 2$

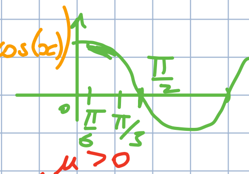
$= 2$

③



③ $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{-\sin(x)}{\cos(x)}\,dx$

① $\dfrac{-\sin(x)}{\cos(x)} \to \ln(\cos(x))$



$\dfrac{u'}{u} \to \ln(u)$, $u > 0$

$\begin{cases} u = \cos(x) \\ u' = -\sin(x) \end{cases}$

② $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{-\sin(x)}{\cos(x)}\,dx = \ln\left(\cos\left(\frac{\pi}{3}\right)\right) - \ln\left(\cos\left(\frac{\pi}{6}\right)\right)$

$= \ln\left(\dfrac{1}{2}\right) - \ln\left(\dfrac{\sqrt{3}}{2}\right)$

$= \ln(1) - \ln(2) - \dfrac{1}{2}\ln(3) + \ln(2)$

$= -\dfrac{1}{2}\ln(3)$

Ex :

① $\displaystyle\int_0^1 (1+x)^4\,dx$

u' ?

① $(1+x)^4 = 1\,(1+x)^4 \to \dfrac{(1+x)^5}{5}$

$u' \times u^n \to \dfrac{u^{n+1}}{n+1}$

$u = 1+x$

$u' = 1$

② $\displaystyle\int_0^1 (1+x)^4\,dx = \frac{(1+1)^5}{5} - \frac{(1+0)^5}{5}$

$= \dfrac{32}{5} - \dfrac{1}{5}$

$= \dfrac{31}{5}$ u.a.

② $\displaystyle\int_2^3 \frac{1}{3t}\,dt =$

$\dfrac{u'}{u} \to \ln(u)$ $\quad u > 0$

$u = 3t$

$u' = 3$

$\dfrac{1}{3t} = \dfrac{1}{3}\,\dfrac{3}{3t} \to \dfrac{1}{3}\ln(3t)$

$\displaystyle\int_2^3 \frac{1}{3t}\,dt = \frac{1}{3}\ln(3\times 3) - \frac{1}{3}\ln(3\times 2)$

$= \dfrac{1}{3}\ln 9 - \dfrac{1}{3}\ln 6$

$= \dfrac{1}{3}\ln\left(\dfrac{9}{6}\right) = \dfrac{1}{3}\ln\left(\dfrac{3}{2}\right)$

③ $\displaystyle\int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \cos(3x)\,dx$